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Ward Identities and Radiative Corrections in QED with Conformal Gauge

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A model of QED with conformally invariant gauge is considered. This gauge, being essentially nonlocal, is written in a local form by means of two nonphysical scalar fields. Using the BRST symmetry and the additional residual symmetry, a system of Ward identities is derived. These Ward identities are applied to prove the renormalizability of the model as well as to investigate the radiative corrections. A new class of conformal anomalies arises, connected with the absence of radiative corrections to the propagators including auxiliary fields.

1. INTRODUCTION

Some years ago conformal models of quantum electrodynamics based on nondecomposable representations of the conformal group were proposed (Binegar *et al.*, 1983*a,b*; Zaikov, 1985; Furlan *et al.*, 1985). Although the difficulties connected with the pure longitudinality of the conformally invariant photon propagator as well as the construction of conformal gauge fixing were avoided, new problems appeared. The most essential problem is that the theory is self-consistent only in the free-field case (zero charge) (Petkova *et al.*, 1985). The same problem also arises (Krasnikov, 1983) for the model proposed in Fradkin *et al.* (1983). The reason this problem arises is that in all these models it is assumed that the electromagnetic potential is transformed according to an irreducible representation of the dilatation subgroup with a canonical dimension. A consequence of the latter assumption is that the total photon propagator is of Adler–Johnson–Baker type, i.e., coincides with the free-field one (Adler, 1972*a,b*; Baker and Johnson, 1979). To avoid the above-mentioned difficulty, Stanev and Todorov (1988) started from the nonvanishing conformal invariant current two-point Wightman function

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and found from the Maxwell equation that the field tensor is also transformed with a nondecomposable representation of the dilatation subgroup. Then the conformally invariant two-point functions for the field tensor contain the log terms appearing in the perturbative theory. However, difficulties connected with the formulation of the theory in terms of electromagnetic potential appear in Stanev and Todorov (1988).

In the present paper a modified model (Zaikov, 1986*a*) of conformal QED is considered in the context of a perturbation approach. We remark that the latter model is also not free from the difficulties associated with the above-mentioned models. Let us recall that this model is modified so that a generalization for the Yang–Mills theory is possible (Zaikov, 1986*b*). It is assumed (Binegar *et al.*, 1983*a,b*; Zaikov, 1985; Furlan *et al.*, 1985; Petkova *et al.*, 1985) that the electromagnetic potential is transformed according to a nondecomposable representation of the conformal group. For this purpose a dimensionless scalar field $R(x)$ is introduced as a fifth component of the electromagnetic potential. The latter permits the existence of a nonzero transverse part of the conformally invariant photon two-point function as well as the construction of a conformal gauge term in the Lagrange approach. In order to include the interaction with matter fields, it is necessary to introduce a second dimensionless scalar field $S(x)$. After integration over these auxiliary fields it is shown [see also Petkova *et al.* (1985) and Zaikov (1986*a*), where this integration is provided only on a formal level] that we have a nonlocal admissible gauge (without Faddeev–Popov ghost fields). As a consequence of the presence of these auxiliary fields, the Lagrangian obeys an additional BRST-like residual symmetry (Zaikov, 1986*a*; Todorov, 1987). It should be pointed out that the latter symmetry is an ordinary one, unlike the BRST symmetry, which is a supersymmetry (Becchi *et al.*, 1976). The BRST-like symmetry is used to derive further Ward identities making it possible to prove also the renormalizability of the theory in the nonphysical sector as well as to investigate the radiative corrections of the two-point functions. It is shown that the two-point function Γ_{AR} is free from radiative correction, although Γ_{AA} has pure transverse corrections as in the ordinary theory. The latter points out that in this case we have a second-class conformal anomaly, because (Binegar *et al.*, 1983*a,b*; Zaikov, 1985; Furlan *et al.*, 1985) the conformal invariance strongly connects the transverse part of Γ_{AA} with Γ_{AR}. We recall that the first-class conformal anomaly arose with the log terms in Γ_{AA}. Another conclusion from the Ward identities is that Γ_{SS} is also free from radiative corrections.

These two types of conformal anomalies break down only if the perturbative sum gives the Adler–Baker–Johnson photon propagator. However, in the latter case, as mentioned above, the electromagnetic interaction breaks down, too.

A similar two types of conformal anomalies appear in the pure Yang–Mills theory (Zaikov, 1991) in conformal gauge. However, in the QED in conformal gauge an additional BRST-like symmetry appears. The BRST-like transformations are nonlinear if fermion fields are included. The price for their linearization is the introduction of an infinite set of composite fields. In order to study the consequences of the BRST-like symmetry for the renormalized Green's functions, it is necessary to prove the renormalizability of the model also in a nonphysical sector where nonzero sources of these composite fields are present.

The paper is organized as follows: In Section 2 the model proposed in Zaikov (1986*a*) is described. In Section 3, BRST (Kugo and Ojima, 1979) and BRST-like symmetries (Zaikov, 1986*a*; Becchi *et al.*, 1976) are considered. These symmetries are applied to derive a system of Ward identities. Additional Ward identities as a consequence of the BRST-like symmetry arise here. In Section 4 these Ward identities are used to prove the renormalizability of the theory and to investigate the radiative corrections of the two-point functions.

2. CONFORMAL GAUGE IN QED

To set the notations, we give a brief review of the model (Zaikov, 1986*a*). The Lagrangian is constructed by adding to the singular Lagrangian of massless QED

$$\mathscr{L}_{\text{QED}} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\hat{\partial} - e\hat{A})\psi \tag{2.1}$$

(where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$) a conformally invariant gauge-fixing term containing two nonphysical massless dimensionless scalar fields R and S:

$$\mathscr{L}_{\text{GF}} = \tfrac{1}{2}\partial^\mu A_\mu \,\Box R + \frac{\alpha}{8}(\Box R)^2 + \frac{\nu}{2}(\Box S)^2 + F_{\mu\nu}\,\partial^\mu R\,\partial^\nu S \tag{2.2}$$

Here α is a gauge-fixing parameter and ν is an arbitrary real parameter. It can be checked that (2.2) is a conformal invariant if all the fields except S are transformed according to a representation of the conformal group defined by

$$\mathscr{D}\Phi(x) = -i(d_\Phi + x^\tau\,\partial_\tau)\Phi(x) \tag{2.3a}$$

$$\mathscr{K}_\mu\Phi(x) = i[2x_\mu(d_\Phi + x^\tau\,\partial_\tau) - x^2\,\partial_\mu + 2ix^\tau\Sigma_{\mu\tau}]\Phi(x) \tag{2.3b}$$

where $\Phi = A, R, \psi$; $\mathscr{D}$, $\mathscr{K}_\mu$, and $\Sigma_{\mu\nu}$ are generators of dilatations, special conformal transformations, and the spin part of the Lorentz transformations, respectively, and d_Φ is the corresponding (canonical) dimension ($d_A = 1$, $d_R = 0$, $d_\psi = \tfrac{3}{2}$). The field S is transformed according to the following conformal law:

$$\begin{aligned} \mathscr{D}S(x) &= -i(x^\tau\,\partial_\tau S(x) - 1) \\ \mathscr{K}_\mu S(x) &= i(2x_\mu x^\tau\,\partial_\tau - x^2\,\partial_\mu)S(x) - 2ix_\mu \end{aligned} \tag{2.4}$$

Let us note that the field A is transformed under the "basic" (homogeneous) conformal law (2.3), but the nonhomogeneous conformal law (2.4) for S makes it possible to construct a conformally invariant gauge fixing in a local form.

Let us consider (for now on a formal level) the functional integral

$$\mathscr{F}(A, h, H) = \int DR\,DS \exp\left[i \int d^4x(\mathscr{L}_{\mathrm{GF}} + hR + HS)\right] \tag{2.5}$$

where h and H are sources of the nonphysical fields R and S, respectively. In the case $\alpha = 0$ the formal integration over R is extremely simple:

$$\mathscr{F}(A, h, H) = \int DS \exp\left\{i \int d^4x\left[\frac{v}{2}(\Box S)^2 + HS\right]\right\} \prod_x \delta(\tfrac{1}{2}\Box\,\partial^\mu A_\mu - \partial^\mu F_{\mu\nu}\,\partial^\nu S + h) \tag{2.6}$$

Then the Faddeev–Popov determinant

$$\dot{\Delta} = \int D\Lambda\,\mathscr{F}(A + \partial\Lambda, h, H)$$

does not depend on A and consequently (2.2) is an appropriate gauge fixing for quantum electrodynamics. In a general gauge ($\alpha \neq 0$) one obtains

$$\mathscr{F}(A, h, H) = \int DS \exp\left\{i \int d^4x\left[\frac{v}{2}(\Box S)^2 + HS - \frac{2}{\alpha}\int d^4y\,\mathscr{C}(x)D_2(x-y)\mathscr{C}(y)\right]\right\} \tag{2.7}$$

where

$$D_2(x) = -\frac{1}{(4\pi)^2}\ln(-\mu^2x^2 + i\varepsilon) \tag{2.8}$$

is the Green's function for the equation $\Box^2 f = 0$, μ is a parameter with dimension of mass, and the notation

$$\mathscr{C}(x) = \tfrac{1}{2}\Box\,\partial^\mu A_\mu(x) - \partial^\nu S(x)(g_{\mu\nu}\Box - \partial_\mu\,\partial_\nu)A^\mu(x) + h(x)$$

is introduced. It is easy to check that this is an appropriate gauge, too. Let us note, however that the integration over the field R breaks the conformal symmetry in the case $\alpha \neq 0$ because a parameter with dimension of mass appears. In the case $\alpha = 0$ one may consider (2.6) as a formal expression only. Indeed, let us write the Green's functional

$$Z(j, h, H, \bar{\chi}, \chi) = \int DA\, D\psi\, D\bar{\psi} \exp\left[i \int d^4x(\mathcal{L}_{\mathrm{QED}} + j^\mu A_\mu + \bar{\chi}\psi + \bar{\psi}\chi)\right] \mathcal{F}(A, h, H) \tag{2.9}$$

Inserting (2.6) into (2.9), one can see that the gauge-fixing condition is too complicated to be solved explicitly and can be used in calculations only if one returns to the integral representation (2.5). So, in both cases we shall understand the (nonlocal) gauge-fixing functional $\mathcal{F}(A, h, H)$ as an integral (2.5).

Turning to a perturbative calculation of Z, one observes that the results are incompatible with the simplest assumptions about the conformal properties of the quantum model. The starting point for a perturbative treatment is a system of free fields. The set of the two-point functions, or, in terms of functional integration, a "measure" in an appropriate functional space, contains all the essential information about this system. The classical Lagrangian of the free bosonic fields

$$\mathcal{L}_{\mathrm{B}}^{(2)}(x) = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2}\, \partial^\mu A_\mu\, \Box R + \frac{\alpha}{8}(\Box R)^2 + \frac{\nu}{2}(\Box S)^2 \tag{2.10}$$

is not invariant with respect to the representation (2.3), (2.4) of the conformal group. However it is invariant with respect to a "nonbasic" conformal law (Zaikov, 1985; Petkova *et al.*, 1985) for the fields A, R. In order to preserve the invariance at the quantum level, one must choose a proper way of integration when calculating the free bosonic Green's functional

$$Z_{\mathrm{B}}^{(0)}(j, h, H) = \int DA\, DR\, DS \exp\left[i \int d^4x(\mathcal{L}_{\mathrm{B}}^{(2)} + j^\mu A_\mu + hR + HS)\right] \tag{2.11}$$

The invariance will be lost in the case $\alpha \neq 0$ if one integrates over R as in (2.7) proceeding to a nonlocal gauge fixing. Let us remark that the field R has been introduced as a fifth component of the potential $\mathcal{A} = (A_\mu, R)$.

Inverting the differential operator $\mathscr{D}$ in the bilinear form

$$\int d^4x\, \mathscr{A}^T \mathscr{D}\mathscr{A} \equiv \int d^4x \left[-\tfrac{1}{4} F^{\mu\nu}_{\mu\nu} + \tfrac{1}{2}\, \partial^\mu A_\mu + \frac{\alpha}{8} (\Box R)^2 \right]$$

one obtains the matrix of propagators. As a reflection of the invariance of $\mathscr{L}^{(2)}_{\mathrm{B}}$, the two-point functions are invariant with respect to a representation of the conformal group which leaves the vacuum invariant and transforms the free fields A and R according to a nonbasic conformal law. The generators $\tilde{K}_\mu$ of special conformal transformations in this representation obey the following commutation relations with the fields A and R:

$$\begin{aligned} [A_\nu(x), \tilde{K}_\mu] &= i[2x_\mu(d_\phi + x^\tau\, \partial_\tau) - x^2\, \partial_\mu] A_\nu(x) \\ &\quad - 2x^\tau(\Sigma_{\mu\tau})^\rho_\nu A_\rho(x) + 2ig_{\mu\nu} R(x) \\ [R(x), \tilde{K}_\mu] &= i(2x_\mu x^\nu\, \partial_\nu - x^2\, \partial_\mu) R(x) \end{aligned} \tag{2.12}$$

where $(\Sigma_{\mu\tau})^\rho_\nu = i(\delta^\rho_\mu g_{\nu\tau} - \delta^\rho_\tau g_{\nu\lambda})$.

The two-point function of the free field S is $(1/i\nu)D_2$ and, as mentioned above, it contains a parameter with the dimension of mass. It is conformally invariant if

$$\begin{aligned} [S(x), \tilde{D}] &= -i(x^\tau\, \partial_\tau S(x) - \hat{q}) \\ [S(x), \tilde{K}_\mu] &= i(2x_\mu x^\tau\, \partial_\tau - x^2\, \partial_\mu) S(x) - 2ix_\mu \hat{q} \end{aligned} \tag{2.13}$$

where $\tilde{D}$ is the generator of dilatations and $\hat{q}$ is a constant operator with the following properties (Sotkov and Stoyanov, 1980, 1983):

$$\langle 0|\hat{q}|0\rangle = \langle 0|\hat{q}\hat{q}|0\rangle = 0$$

$$\langle 0|S(x)\hat{q}|0\rangle = \text{const}$$

The two-point functions of the bosonic fields are also conformally invariant with respect to another representation. It transforms the fields according to a law coinciding with the one for the classical fields [see (2.3), (2.4)]. The generators D and K_μ of dilatations and special conformal transformations do not annihilate the vacuum state,

$$D|0\rangle = U|0\rangle \neq 0$$

$$K_\mu|0\rangle = V|0\rangle \neq 0$$

Here U and V_μ are operators obeying the commutation relations

$$\begin{aligned}
&[S(x), U]=i(1-\hat{q}) \\
&[S(x), V_\mu]=2ix_\mu(\hat{q}-1) \\
&[A_\lambda(x), U]=0 \\
&[A_\lambda(x), V_\mu]=2ig_{\lambda\mu}R(x)
\end{aligned} \tag{2.14}$$

The field R commutes with U and V_μ. It is obvious that the generators $\tilde{D}$, $\tilde{K}_\mu$ of the nonbasic representation may be written as

$$\tilde{D}=D-U$$

$$\tilde{K}_\mu=K_\mu-V_\mu$$

It is known (Zaikov, 1985; Petkova *et al.*, 1985) that the two-point function of the free A is not purely longitudinal. It is clear that there is no contradiction between this statement and the conformal properties of the free fields. Unfortunately, because of two kinds of anomalies, the simple conformal properties are lost when perturbative corrections are taken into account.

3. RESIDUAL SYMMETRY AND WARD IDENTITIES

It is convenient to redefine the fields A, R:

$$A_\mu \to \frac{1}{e} A_\mu, \qquad R \to veR$$

in order to make symmetry transformations independent of the parameters e, v. Let us remark that the conformally invariant Lagrangian (rewritten in terms of the new fields A, R)

$$\begin{aligned}
\mathscr{L}_{\mathrm{QED}}+\mathscr{L}_{\mathrm{GF}}=&-\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}+\frac{v}{2}\partial_\mu A^\mu \,\Box R+\frac{\alpha'}{8}(\Box R)^2 \\
&+\frac{v}{2}(\Box S)^2+vF_{\mu\nu}\,\partial^\mu R\,\partial^\nu S+\bar{\psi}(i\hat{\partial}-\hat{A})\psi
\end{aligned} \tag{3.1}$$

where $\alpha'=v^2e^2\alpha$, is not invariant with respect to the transformations $A\to A+\partial\Lambda$, because of the corresponding gauge fixing. However it obeys some residual symmetry, which becomes clear if a free Faddeev–Popov ghost field Lagrangian is included:

$$\mathscr{L}_{\mathrm{GH}}=\frac{v}{2}\bar{\eta}\,\Box^2\eta \tag{3.2}$$

where η is a dimensionless scalar Faddeev–Popov ghost field transformed under the same conformal law as the field S. Let us remark that for convenience a generalized free ghost field is introduced (Zaikov, 1986*a*) here ($\Box^2\eta = 0$) instead of the free ghost field in the ordinary QED.

Then it is easy to check that the Lagrangian (3.1) is invariant with respect to the following BRST transformations:

$$\begin{aligned} &\delta B = \delta\lambda\, \partial\eta(x) \\ &\delta\psi = -i\,\delta\lambda\, \eta(x)\psi(x) \\ &\delta\bar{\psi} = i\,\delta\lambda\, \bar{\psi}(x)\bar{\eta}(x) \\ &\delta R = \delta S = \delta\eta = 0 \\ &\delta\bar{\eta} = -\tfrac{1}{2}\delta\lambda\, R(x) \end{aligned} \tag{3.3}$$

where $\delta\lambda$ is an infinitesimal odd Grassmann parameter.

Because of the existence of the nonphysical fields, the Lagrangian

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{QED}} + \mathscr{L}_{\text{GF}} + \mathscr{L}_{\text{GH}}$$

obeys also the following residual (BRST-like) symmetry:

$$\begin{aligned} &\Delta B_\mu = \Delta\varepsilon\, \partial_\mu S(x) \\ &\Delta R = 0 \\ &\Delta S = -\frac{\Delta\varepsilon}{2} R(x) \\ &\Delta\psi = -i\,\Delta\varepsilon\, S(x)\psi(x) = -i\,\Delta\varepsilon\, \psi_1(x) \\ &\Delta\bar{\psi} = i\,\Delta\varepsilon\, S(x)\bar{\psi}(x) = i\,\Delta\varepsilon\, \bar{\psi}_1(x) \\ &\Delta\eta = \Delta\bar{\eta} = 0 \end{aligned} \tag{3.4}$$

where $\Delta\varepsilon$ is an infinitesimal (even) parameter.

The transformations (3.4), like the BRST ones, are nilpotent only in the bosonic sector. However, in the fermionic sector this is not the case. Then for our purpose it is convenient to introduce composite fields $\psi_k(x)$, $\bar{\psi}_k(x)$ defined by

$$\begin{aligned} &\Delta\psi_k = -i\,\Delta\varepsilon\, \psi_{k+1}(x) \\ &\Delta\bar{\psi}_k = i\,\Delta\varepsilon\, \bar{\psi}_{k+1}(x), \qquad k = 0, 1, \ldots, \qquad \psi_0 = \psi, \quad \bar{\psi}_0 = \bar{\psi} \end{aligned} \tag{3.4a}$$

The explicit form of the lowest ψ_k is given by

$$\begin{aligned} &\psi_0 = \psi, \qquad \psi_1 = S\psi, \qquad \psi_2 = S^2\psi - \frac{i}{2} R\psi, \qquad \psi_3 = S^3\psi - \frac{3i}{2} RS\psi \\ &\psi_4 = S^4\psi - 3iRS\psi - \tfrac{3}{4}R^2\psi, \qquad \psi_5 = S^5\psi - RS^3\psi - \tfrac{15}{4}R^2 S\psi, \ldots \end{aligned} \tag{3.5}$$

We remark that the Lagrangian (3.1) depends on the fields R and S only through their derivatives and consequently it is invariant also with respect to the following transformation laws:

$$\begin{aligned} &\Delta_1 R(x) = \Delta\varepsilon_1 \\ &\quad \Delta_1 A = \Delta_1 S = \cdots = \Delta_1 \psi = \Delta_1 \psi_1 = \Delta_1 \bar{\psi}_1 = 0 \\ &\Delta_2 S(x) = \Delta\varepsilon_2 \\ &\quad \Delta_2 A = \Delta_2 R = \cdots = \Delta_2 \psi = \Delta_2 \bar{\psi} = 0 \end{aligned} \tag{3.6}$$

where $\varepsilon_{1,2}$ are constant parameters.

It is not difficult to check that with respect to the transformations (3.6) the composite fields (3.5) obey the following transformation laws:

$$\begin{aligned} &\Delta_1 \psi_k = -\Delta\varepsilon_1 \frac{ik(k-1)}{4} \psi_{k-2} \\ &\Delta_1 \bar{\psi}_k = \Delta\varepsilon_1 \frac{ik(k-1)}{4} \bar{\psi}_{k-2}, \qquad k = 2, \ldots \end{aligned} \tag{3.7a}$$

$$\Delta_2 \psi_k = \Delta\varepsilon_2\, k\psi_{k-1}, \qquad \Delta_2 \bar{\psi}_k = \Delta\varepsilon_2\, k\bar{\psi}_{k-1}, \qquad k = 1, \ldots \tag{3.7b}$$

Now, let us consider the following Green's functional:

$$\begin{aligned} &Z(j, h, H, \chi, \bar{\chi}, \kappa_k, \bar{\kappa}_k, \rho, \bar{\rho}) \\ &\quad = \exp G(j, h, H, \chi, \bar{\chi}, \kappa_k, \bar{\kappa}_k, \rho, \bar{\rho}) \\ &\quad = \int \mathscr{D} A \mathscr{D} R \mathscr{D} S \mathscr{D} \psi \bar{\mathscr{D}} \bar{\psi} \mathscr{D}\eta \mathscr{D}\bar{\eta} \exp\left\{ i \int d^4x \left[\mathscr{L}_{\text{eff}} + jA + hR \right.\right. \\ &\qquad \left.\left. + HS + \sum_{k=1}^{\infty} (\bar{\psi}_k \chi_k + \bar{\chi}_k \psi_k) + \bar{\psi}\kappa + \bar{\kappa}\psi + \bar{\eta}\rho + \bar{\rho}\eta \right] \right\} \end{aligned} \tag{3.8}$$

Taking into account that $\mathscr{L}_{\text{eff}}$ is invariant with respect to the BRST transformations (3.3), the BRST-like transformations (3.4), and the transformations (3.6), we derive the following Ward identities:

$$\begin{aligned} &\int \mathscr{D}A \ldots \mathscr{D}\bar{\eta} \exp i \int d^4x \{ \mathscr{L}_{\text{eff}} + jA + \cdots + \rho\bar{\eta} \} \\ &\quad \times \int d^4x (j\delta A + h\delta R + H\delta S + \delta\bar{\psi}\kappa + \kappa\delta\bar{\psi} \\ &\quad + \delta\bar{\psi}_k \chi_k + \bar{\chi}_k \delta\psi_k + \delta\bar{\eta}\rho + \bar{\rho}\delta\eta) = 0 \end{aligned} \tag{3.9a}$$

$$\int \mathscr{D}A \ldots \mathscr{D}\bar{\eta} \exp i \int d^4x\{\mathscr{L}_{\text{eff}}+jA+\cdots+\rho\bar{\eta}\}$$
$$\times \int d^4x(j\Delta A+H\Delta S+\Delta\bar{\psi}\chi+\chi\Delta\bar{\psi}+\Delta\bar{\psi}_k\chi_k$$
$$+\bar{\chi}_k\Delta\psi_k+\Delta\bar{\eta}\rho+\bar{\rho}\Delta\eta)=0 \tag{3.9b}$$

$$\int \mathscr{D}A_\mu \ldots \mathscr{D}\bar{\eta} \exp i \int d^4x\{\mathscr{L}_{\text{eff}}+jA+\cdots+\rho\bar{\eta}\}$$
$$\times \int d^4x(h\Delta_1 R+\Delta_1\bar{\psi}_k\chi_k+\bar{\chi}_k\Delta_1\psi_k)=0 \tag{3.9c}$$

$$\int \mathscr{D}A_\mu \ldots \mathscr{D}\bar{\eta} \exp i \int d^4x\{\mathscr{L}_{\text{eff}}+jA+\cdots+\rho\bar{\eta}\}$$
$$\times \int d^4x(H\Delta_2 S+\Delta_2\bar{\psi}_k\chi_k+\bar{\chi}_k\Delta_2\psi_k)=0 \tag{3.9d}$$

where summation over repeating indexes $k=0, 1, \ldots$ is implicit. The analysis of the Ward identities (3.7) is simpler for the one-particle irreducible Green's functions:

$$\Gamma(A, R, S, \psi, \bar{\psi}, \eta, \bar{\eta}, \chi_n, \bar{\chi}_n)$$
$$=-iG(j, h, H, \chi, \bar{\chi}, \chi_n, \bar{\chi}_n, \rho, \bar{\rho})$$
$$-\int d^4x(jA+hR+HS+\bar{\psi}\chi+\bar{\chi}\psi+\bar{\eta}\rho+\bar{\rho}\eta) \tag{3.10}$$

Now, taking into account that

$$j^\mu=-\frac{\delta\Gamma}{\delta A_\mu}, \qquad h=-\frac{\delta\Gamma}{\delta R}, \qquad H=-\frac{\delta\Gamma}{\delta S}$$
$$\bar{\chi}=\frac{\delta\Gamma}{\delta\psi}, \qquad \chi=-\frac{\delta\Gamma}{\delta\bar{\psi}}, \qquad -i\frac{\delta G}{\delta\chi_n}=\frac{\delta\Gamma}{\delta\chi_n} \tag{3.11}$$
$$\bar{\rho}=\frac{\delta\Gamma}{\delta\eta}, \qquad \rho=-\frac{\delta\Gamma}{\delta\bar{\eta}}, \qquad -i\frac{\delta G}{\delta\bar{\chi}_n}=\frac{\delta\Gamma}{\delta\bar{\chi}_n}, \qquad n=1, 2, \ldots$$

we rewrite the Ward identities in the following form:

$$\int d^4x \left\{ \left[\partial_\mu \frac{\delta \Gamma}{\delta A_\mu} + i\left(\frac{\delta \Gamma}{\delta \psi} \psi + \bar{\psi} \frac{\delta \Gamma}{\delta \bar{\psi}} \right) + i \sum_{k=1}^{\infty} \left(\frac{\delta \Gamma}{\delta \chi_k} \chi_k + \bar{\chi}_k \frac{\delta \Gamma}{\delta \bar{\chi}_k} \right) \right] \eta(x) + \tfrac{1}{2} R \frac{\delta \Gamma}{\delta \bar{\eta}} \right\} = 0 \tag{3.12a}$$

$$\int d^4x \left[S(x)\, \partial_\mu \frac{\delta \Gamma}{\delta A_\mu} + \tfrac{1}{2} R(x) \frac{\delta \Gamma}{\delta S} - i\left(\frac{\delta \Gamma}{\delta \psi} \frac{\delta \Gamma}{\delta \bar{\chi}_1} - \frac{\delta \Gamma}{\delta \chi_1} \frac{\delta \Gamma}{\delta \bar{\psi}} \right) - ie \sum_{k=1}^{\infty} \left(\bar{\chi}_k \frac{\delta \Gamma}{\delta \bar{\chi}_{k+1}} + \frac{\delta \Gamma}{\delta \chi_{k+1}} \chi_k \right) \right] = 0 \tag{3.12b}$$

$$\int d^4x \left[\frac{\delta \Gamma}{\delta R} + \frac{i}{2} (\bar{\chi}_2 \psi - \psi_2 \bar{\chi}) + \frac{i}{4} \sum_{k=2}^{\infty} k(k-1) \left(\bar{\chi}_k \frac{\delta \Gamma}{\delta \bar{\chi}_{k-2}} + \frac{\delta \Gamma}{\delta \chi_{k-2}} \chi_k \right) \right] = 0 \tag{3.12c}$$

$$\int d^4x \left[\frac{\delta \Gamma}{\delta S} - \bar{\chi}_1 \psi - \bar{\psi} \chi_1 - \sum_{k=2}^{\infty} k \left(\bar{\chi}_k \frac{\delta \Gamma}{\delta \bar{\chi}_{k-1}} - \frac{\delta \Gamma}{\delta \chi_{k-1}} \chi_k \right) \right] = 0 \tag{3.12d}$$

The fields R and S in (3.10) are interacting fields, which is not the case for the ghost field included in (3.12a). Moreover, in (3.12b) a term that is nonlinear with respect to Γ appears, as in the Slavnov–Taylor identities in the Yang–Mills theory.

4. RENORMALIZABILITY OF THE QED IN CONFORMAL GAUGE

Throughout the previous section we assumed that the dimensional regularization is carried out. This means that the divergent terms arise only in the limit $D \to 4$. We remark that the effective Lagrangian and the measure are invariant with respect to the BRST transformations (3.3) and the transformations (3.4) for arbitrary D-dimensional space-time. Consequently, the Ward identities (3.10) are satisfied for any D-dimensional space-time. The latter allows us to provide a gauge-invariant minimal renormalization which consists of the subtraction of the terms divergent with $D \to 4$. Let us consider first the bosonic part of the effective action:

$$\Gamma_B = \Gamma_{|\psi = \bar{\psi} = \chi_k = \bar{\chi}_k = 0}$$

Taking into account that the ghost field is free and using (3.12a), one obtains

$$\partial_\mu \frac{\delta\Gamma_B}{\delta A_\mu} + \frac{\nu}{2} \Box^2 R = 0 \tag{4.1a}$$

and the Ward identities (3.12b)–(3.12d) yield

$$\int d^4x \left(S\,\partial_\mu \frac{\delta\Gamma_B}{\delta A_\mu} + \frac{1}{2} R \frac{\delta\Gamma_B}{\delta S} \right) = 0 \tag{4.1b}$$

$$\int d^4x \frac{\delta\Gamma_B}{\delta R} = 0 \tag{4.1c}$$

$$\int d^4x \frac{\delta\Gamma_B}{\delta S} = 0 \tag{4.1d}$$

It is clear [see equations (4.1c), (4.1d)] that Γ_B may be expressed in terms of the field A and the derivatives $\partial_\mu R$, $\partial_\mu S$ of the scalar fields. Then solving (4.1a), (4.1b), one obtains

$$\Gamma_B = \int d^4x \left[\frac{\nu}{2} (\Box S)^2 + \frac{\nu}{2} \partial^\mu A_\mu \Box R \right] + \Gamma'_B(F_{\mu\nu}, \partial_\mu R, \partial_\mu S) \tag{4.2}$$

It follows from (4.1b) that there is an expression for Γ'_B in terms of $F_{\mu\nu}$, $f_{\mu\nu} = \partial_\mu R\, \partial_\nu S - \partial_\mu S\, \partial_\nu R$, and the derivative $\partial_\mu R$. Looking for divergent terms in Γ'_B, one finds that the bosonic counterterm, which must be added to the Lagrangian, is a linear combination of the following quantities:

$$F_{\mu\nu}F^{\mu\nu}, \qquad F_{\mu\nu}f^{\mu\nu}, \qquad f_{\mu\nu}f^{\mu\nu}, \qquad \partial_\mu R\, \partial^\mu R$$

$$\partial_\mu R\, \partial^\mu R\, \Box R, \qquad \Box R\, \Box R, \qquad (\partial_\mu R\, \partial^\mu R)^2$$

Let us remark that S is a quasifree field:

$$\left. \frac{\delta^2\Gamma_B}{\delta S(x)\, \delta S(y)} \right|_0 = \nu \Box^2 \delta(x-y) \tag{4.3a}$$

$$\left. \frac{\delta^2\Gamma_B}{\delta A_\mu(x)\, \delta S(y)} \right|_0 = 0 \tag{4.3b}$$

$$\left. \frac{\delta^2\Gamma_B}{\delta R(x)\, \delta S(y)} \right|_0 = 0 \tag{4.3c}$$

($|_0$ means $A = R = S = \psi = \bar{\psi} = 0$). The field S is an interacting one, but its propagator coincides with the free propagator.

Writing down all admissible counterterms containing fermionic fields and taking into account the constraints on them, following from the linear

Ward identities (3.12a), (3.12c), and (3.12d), one finds that the sum of the counterterms can be written as

$$
\begin{aligned}
I^{\mathrm{ct}} = \int d^4x \Big\{ & -\frac{Z_3-1}{4e^2} F_{\mu\nu}F^{\mu\nu} + (Z_1-1)\bar{\psi}(i\hat{\partial}-\hat{A})\psi + (Z_4-1)\frac{\alpha'}{8}(\Box R)^2 \\
& + (Z_5-1)\nu F_{\mu\nu}\,\partial^\mu R\,\partial^\nu S + \tfrac{1}{2} Y_6\,\partial_\mu R\,\partial^\mu R + Y_7\,\partial_\mu R\,\partial^\mu R\,\Box R \\
& + Y_8(\partial_\mu R\,\partial^\mu R)^2 + Y_9 f_{\mu\nu} f^{\mu\nu} + Y_{10}\bar{\psi}\,\hat{\partial} R\psi + Y_{11}\bar{\psi}\,\hat{\partial} S\psi \\
& + \sum_{k=1}^{\infty} [g_k(R,S)\bar{\chi}_k\psi + g_k^*(R,S)\psi\bar{\chi}_k] \Big\}
\end{aligned}
\tag{4.4}
$$

where Z_1, $Z_3, \ldots, Z_5$, and $Y_6, \ldots, Y_{11}$ are divergent constants and $g_k(R,S)$ and $g_k^*(R,S)$ $(k=1,2,\ldots)$ are functions with divergent coefficients. The constant Y_6 is dimensional and the renormalized Lagrangian is not conformally invariant. Introducing a loop expansion parameter λ, one can write

$$
\begin{aligned}
Z_i &= 1 + o(\lambda), \qquad i = 1, 3, 4, 5 \\
Y_k &= o(\lambda), \qquad k = 6, \ldots, 11
\end{aligned}
$$

As usual, $f(\lambda) = o(\lambda^n)$ means $\lim_{\lambda\to 0} \lambda^{-n} f(\lambda) = 0$ and $f(\lambda) = \mathcal{O}(\lambda^n)$ means that $|\lambda^{-n} f(\lambda)|$ remains bounded when $\lambda \to 0$.

The functions $g_k(R,S)$ obey two systems of equations:

$$
\begin{aligned}
&\frac{\partial g_1}{\partial R} = 0, \qquad \frac{\partial g_2}{\partial R} = 0 \\
&\frac{\partial g_k}{\partial R} + ik(k-1)g_{k-2} = 0, \qquad k = 3, 4, \ldots
\end{aligned}
\tag{4.5}
$$

and

$$
\begin{aligned}
&\frac{\partial g_1}{\partial S} = 0 \\
&\frac{\partial g_k}{\partial R} - k g_{k-1} = 0, \qquad k = 2, 3, \ldots
\end{aligned}
\tag{4.6}
$$

The corresponding equations for $g_k^*(R,S)$ may be obtained by complex conjugation.

The nonlinear Ward identity (3.12b) yields additional constraints on g_k, g_k^*. Following the recursive method (see, e.g., Itzykson and Zuber, 1980,

p. 599), we introduce a bilinear operation

$$\Gamma_1 * \Gamma_2 = \int d^4x \left(\frac{\delta \Gamma_1}{\delta \psi} \frac{\delta \Gamma_2}{\delta \bar{\chi}_1} - \frac{\delta \Gamma_1}{\delta \chi_1} \frac{\delta \Gamma_2}{\delta \bar{\psi}} \right)$$

and a linear operator

$$L(\Gamma) = \int d^4x \left[\sum_{k=1}^{\infty} \left(\bar{\chi}_k \frac{\delta \Gamma}{\delta \bar{\chi}_{k+1}} + \frac{\delta \Gamma}{\delta \chi_{k+1}} \chi_k \right) \right]$$

The identity (3.12b) may be written as

$$\Gamma * \Gamma + L(\Gamma) = 0 \tag{4.7}$$

Denoting by Γ^0 and Γ^1 the first and the second terms, respectively, in the loop expansion of Γ, one writes

$$\Gamma = \Gamma^0 + \Gamma^1 + o(\lambda)$$

Equation (4.7) is satisfied by the classical action

$$I = \int d^4x \, \mathscr{L}_{\text{eff}}$$

and therefore Γ^1 obeys the linear equation

$$I * \Gamma^1 + \Gamma^1 * I + L(\Gamma^1) = 0 \tag{4.8}$$

It is enough if the one-loop counterterm I^1_{ct} obeys the weaker condition:

$$I * \Gamma^1_{\text{ct}} + \Gamma^1_{\text{ct}} * I + L(\Gamma^1_{\text{ct}}) = o(\lambda^2) \tag{4.9}$$

Inserting the expression (4.4) for I^1_{ct} into (4.9), we find constraints for the one-loop quantities $Y^{(1)}_{11}$, $g^{(1)}_1$, $g^{*(1)}_1$:

$$Y^{(1)}_{11} = 0, \qquad g^{(1)}_1 = g^{*(1)}_1 = \text{const } [= o(\lambda)] \tag{4.10}$$

and recursive relations

$$\begin{aligned} &g^{(1)}_{k+1}(R, S) - S g^{(1)}_k(R, S) + \frac{i}{2} R \frac{\partial g^{(1)}_k}{\partial S}(R, S) \\ &\quad - g^{(1)}_1 \Omega_k(R, S) = o(\lambda^2) \\ &g^{*(1)}_{k+1}(R, S) - S g^{*(1)}_k(R, S) - \frac{i}{2} R \frac{\partial g^{*(1)}_k}{\partial S}(R, S) \\ &\quad - g^{*(1)}_1 \Omega^*_k(R, S) = o(\lambda^2) \end{aligned} \tag{4.11}$$

where Ω_k, Ω_k^* are functions defined by the equations

$$\psi_k=\Omega_k(R, S)\psi, \qquad \bar{\psi}_k=\Omega_k^*(R, S)\bar{\psi} \tag{4.12}$$

The crucial point in the recursive procedure is whether the one-loop renormalized action $I_1=I+I_{\mathrm{ct}}^1$ obeys the Ward identity (4.7). Introducing the notations

$$\omega_k=\Omega_k+g_k^{(1)}, \qquad \omega_k^*=\Omega_k^*+g_k^{*(1)} \tag{4.13}$$

and taking into account (4.4), (4.11), one can write

$$\begin{aligned} I_1=\int d^4x\bigg\{&\frac{Z_3^{(1)}}{4e^2}F_{\mu\nu}F^{\mu\nu}+Z_1^{(1)}\bar{\psi}(i\hat{\partial}-\hat{A})\psi+Z_4^{(1)}\frac{\alpha'}{8}(\Box R^2)\\ &+Z_5^{(1)}vF_{\mu\nu}\,\partial^\mu R\,\partial^\nu S+\tfrac{1}{2}Y_6^{(1)}\,\partial_\mu R\,\partial^\mu R+Y_7^{(1)}\,\partial_\mu R\,\partial^\mu R\,\Box R\\ &+Y_8^{(1)}(\partial_\mu R\,\partial^\mu R)^2+Y_9^{(1)}f_{\mu\nu}f^{\mu\nu}+Y_{10}^{(1)}\bar{\psi}\,\hat{\partial}R\psi\\ &+\sum_{k=1}^{\infty}[\omega_k(R, S)\bar{\chi}_k\psi+\omega_k^*(R, S)\bar{\psi}\chi_k]\bigg\} \end{aligned} \tag{4.14}$$

where $Z_1^{(1)}, Z_3^{(1)},\dots, Z_5^{(1)}$, and $Y_6^{(1)},\dots, Y_{10}^{(1)}$ are one-loop renormalization constants. It is easy to check that (4.14) is a solution of (4.7) if

$$\begin{aligned} &\omega_1(R, S)=\Omega_1+C^{(1)}=S+C^{(1)}, \qquad C^{(1)}=\text{const}\\ &\omega_{k+1}(R, S)=\omega_1(R, S)\omega_k(R, S)-\frac{i}{2}R\frac{\partial\omega_k}{\partial S}(R, S)\\ &\omega_1^*(R, S)=\Omega_1^*+C^{(1)}=S+C^{(1)}\\ &\omega_{k+1}^*(R, S)=\omega_1^*(R, S)\omega_k^*(R, S)+\frac{i}{2}R\frac{\partial\omega_k^*}{\partial S}(R, S) \end{aligned} \tag{4.15}$$

The solution is

$$\begin{aligned} &\omega_k(R, S)=\Omega_k(R, S+C^{(1)})\\ &\omega_k^*(R, S)=\Omega_k^*(R, S+C^{(1)}) \end{aligned} \tag{4.16}$$

where $C^{(1)}$ does not depend on the fields. It is not difficult to check that if $C^{(1)}=\lambda r$ (r does not depend on λ), then equations (4.11) are satisfied. So the continuation of the recursive procedure is possible (to any order in the expansion on λ) and we find that the Ward identities guarantee the renormalizability of the model (also in the case when sources of the composite fields ψ_k, $\bar{\psi}_k$ are present). There are 10 renormalization constants, $Z_1, Z_3,\dots, Z_5$, $Y_6,\dots, Y_{10}$, and C. The renormalized effective action Γ^{ren} obeys the Ward identities (3.12).

Let us now consider the consequences of this statement for the Green's functions. By differentiation of (3.12a) with respect to $\eta(x)$, $\psi(y)$, and $\bar{\psi}(z)$, we derive the well-known Ward identity:

$$\partial_\mu \left.\frac{\delta^3\Gamma^{\text{ren}}}{\delta A_\mu(x)\,\delta\bar{\psi}(y)\,\delta\psi(z)}\right|_0 = i\left.\left(\delta^4(x-z)\,\frac{\delta^2\Gamma^{\text{ren}}}{\delta\bar{\psi}(x)\,\delta\psi(y)} - \delta^4(x-y)\,\frac{\delta^2\Gamma^{\text{ren}}}{\delta\bar{\psi}(x)\,\delta\psi(y)}\right)\right|_0 \tag{4.17}$$

The same result can be obtained also from (3.12b) by differentiation with respect to S, $\bar{\psi}$, and ψ and replacing the derivatives

$$\frac{\delta^3\Gamma^{\text{ren}}}{\delta S(x)\,\delta\bar{\psi}(y)\,\delta\psi(z)}, \qquad \frac{\delta^3\Gamma^{\text{ren}}}{\delta S(x)\,\delta\bar{\psi}(y)\,\delta\chi_1(z)}$$

with these determined from equation (3.12d).

Now differentiating (3.12b) with respect to R and S, we find

$$\partial_\mu \left.\frac{\delta^2\Gamma^{\text{ren}}}{\delta A_\mu(x)\,\delta R(y)}\right|_0 + \frac{1}{2}\left.\frac{\delta^2\Gamma^{\text{ren}}}{\delta S(x)\,\delta S(y)}\right|_0 = 0 \tag{4.18}$$

Then, taking into account (4.3a), we conclude that the radiative corrections in the two-point vertex Γ_{AR} are also absent.

Now we are able to conclude that if the radiative corrections appear in the two-point photon vertex function, then, according to (4.3a) and (A.2) of the Appendix, a second-class conformal anomaly appears. We recall that we have a first-class conformal anomaly when the coefficient c in (A.1) is equal to a sum of log terms. In this case, if

$$a=b=c \tag{4.19}$$

then the vertex (A.1) is invariant with respect to the representations, which are nondecomposable also with respect to the dilatation subgroup (Dell'Antonio, 1972; Furlan *et al.*, 1985; Zaikov, 1988). However, according to (4.5), here $a=b=1$ and consequently here a second-class conformal anomaly arises connected with the transversality of the radiative corrections.

5. CONCLUSION

We have considered a model, proposed in Zaikov (1986*a*), of conformal QED with two auxiliary massless scalar fields. It is shown that in addition to the ordinary BRST symmetry, the model obeys a residual BRST-like symmetry as well as a symmetry with respect to translations by constants of the auxiliary fields. Ward identities corresponding to these symmetries are obtained, and using them, it is shown that the model is renormalizable. There are ten renormalization constants: nine of them are multiplicative and one is additive. Five types of counterterms arise which are not present in the initial Lagrangian. One of the renormalization constants is dimensional; hence the conformal invariance of the renormalized Lagrangian breaks down. It is shown that the radiative corrections of the two-point vertex function of the electromagnetic potential are purely transverse, although the radiative corrections of the two-point function including the auxiliary field R are absent. The latter points out the existence of a second-class conformal anomaly.

APPENDIX

Let us write down the two-point vertex function for the five-component potential $\mathscr{A}=(A, R)$:

$$\Gamma_{\mathscr{A}\mathscr{A}}=\begin{pmatrix} (\alpha/4)(p^2)^2 & (ia/2)p_\nu p^2 \\ -(ib/2)p_\mu p^2 & c(g_{\mu\nu}p^2-p_\mu p_\nu) \end{pmatrix} \tag{A.1}$$

where α, a, b, and c in general are arbitrary functions of the momentum. We remark that (A.1) is invariant with respect to the conformal transformations (2.12) only if

$$a=b=c=\text{const} \tag{A.2}$$

(Zaikov, 1985; Furlan *et al.*, 1985).

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